Online Appendix for:

Expectations and Learning from Prices

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Contents

B Supplementary Derivations

B.1 Misinference from Assuming the World is Rational

In this section we illustrate a specific example where we consider the implications of misinference in models of wrong Bayesian updating. As explained in Section 3, inferring the correct information from prices when wrong Bayesian updating is pervasive implies that agents must fully understand other agents' mistakes, while failing to recognize those same mistakes in themselves. For example, consider agents who have diagnostic expectations, as in Bordalo et al. (2018). For these agents to correctly infer information from endogenous outcomes, they must fully understand that all other agents are diagnostic. If instead agents fail to recognize their same (to them unknown) mistake in others, then agents infer information as if they lived in a rational world, and this leads to misinference. The same is true of setups with overconfidence, as in Odean (1998), or if some agents are inattentive.

To study the implications of this type of misinference, consider a setup similar to our benchmark framework, but assume instead that the misspecified and true price functions are given by:

$$P^{Mis}(\tilde{s}) = \frac{\tau_s}{\tau_s + \tau_0}\tilde{s} + \frac{\tau_0}{\tau_s + \tau_0}\left(\mu_0 - AZ\tau_0^{-1}\right) \tag{B.1}$$

$$P^{True}(s, \tilde{s}) = \frac{\phi\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}s + \frac{(1-\phi)\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}\tilde{s} + \frac{\tau_0}{\tilde{\tau}_s + \tau_0}\left(\mu_0 - AZ\tau_0^{-1}\right) \tag{B.2}$$

where $\tilde{\tau}_s \neq \tau_s$. For example, when $\tilde{\tau}_s > \tau_s$, this true price function corresponds to the case where both I and U agents trade using a precision that is too high relative to the true precision of the signal, as would arise if agents were diagnostic or if they were over-confident about the precision of new information.

The dotted line in Figure 2 depicts this scenario. In this case $\hat{\alpha} + \hat{\beta} = \frac{\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}$, and changes in $\hat{\alpha} + \hat{\beta}$ along the $x-$axis come from varying $\tilde{\tau}_s$. Here we provide an example of how combining wrong Bayesian updating with misinference can be so detrimental so as to give rise to instability, something which could not arise if we had wrong Bayesian updating alone. Specifically, an immediate application of Proposition 3 tells us that the equilibrium is unstable if and only if:

$$\frac{\tau_s}{\tau_s + \tau_0} < \frac{(1-\phi)\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}. \tag{B.3}$$

When $(1-\phi)\tau_0 - \phi\tau_s > 0$, this condition reduces to:

$$\frac{\tilde{\tau}_s}{\tau_s} > \frac{\tau_0}{(1-\phi)\tau_0 - \phi\tau_s} \tag{B.4}$$

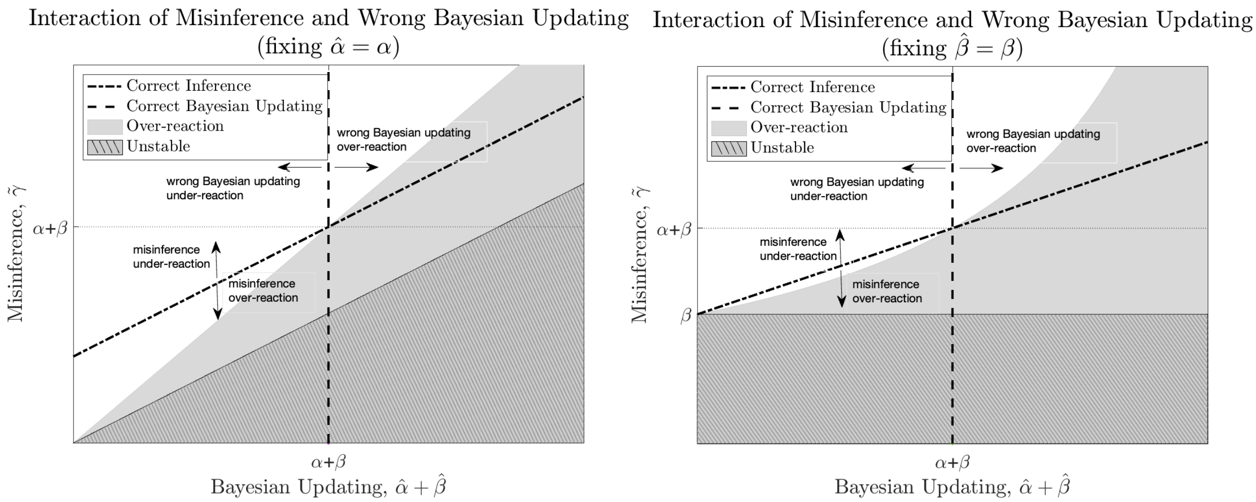
Therefore, for a high enough $\tilde{\tau}_s$, misinference due to the fact that agents believe in a rational world when it is not, can indeed lead to instability. This is shown on the part of the light dotted line in Figure 2 which is in the shaded unstable region.

The intuition behind this example is as follows. Since uninformed agents fail to realize that everyone is over-reacting in the form of $\tilde{\tau}_s > \tau_s$, they extract information from prices using a misspecified model of the world that is less responsive to new information than the true model of the world. This leads them to extract a signal which is more extreme than in reality. Having extracted this extreme signal, uninformed agents then over-react themselves, leading to even further mispricing. Finally, notice that while informed agents are using the correct signal so in a way their over-reaction contributes to stabilizing the equilibrium, their over-reaction also worsens uninformed agents' misinference.

Finally, notice that incorrect bayesian updating can occur along two dimensions: $\hat{\alpha} \neq \alpha$ and $\hat{\beta} \neq \beta$, so that there really are two degrees of freedom in determining $\hat{\alpha} + \hat{\beta}$. In Figure 2 we specified a particular way of varying $\hat{\alpha} + \hat{\beta}$: in the example considered above, all agents are over-confident and varying $\tilde{\tau}_s$ changes both $\hat{\alpha}$ and $\hat{\beta}$, so that $\hat{\alpha} + \hat{\beta} = \frac{\phi\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0} + \frac{(1-\phi)\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0} = \frac{\tilde{\tau}_s}{\tilde{\tau}_s + \tau_0}$. However, $\hat{\alpha}$ and $\hat{\beta}$ need not be linked in this specific way. For example, in Figure 3 we perform a similar exercise, with a different type of variation in $\hat{\alpha} + \hat{\beta}$: the left panel fixes $\hat{\alpha}$, so that all variations in $\hat{\alpha} + \hat{\beta}$ come from changes in $\hat{\beta}$, and the right panel fixes $\hat{\beta}$, so that all variations in $\hat{\alpha} + \hat{\beta}$ come from changes in $\hat{\alpha}$.[1]

[1] In general, $\hat{\alpha}$ and $\hat{\beta}$ are likely to be linked. For example, if informed agents were rational and only uninformed agents were over-confident, this would still lead to $\hat{\alpha} \neq \alpha$, as both $\hat{\alpha}$ and $\hat{\beta}$ are scaled by the average precision across agents. Therefore, fixing $\hat{\alpha}$ and letting $\hat{\beta}$ vary does *not* necessarily amount to fixing the bias of I agents and letting the bias of U agents vary. We show these cases for completeness and for intuition.

Figure 3: Instability, Under- and Over-reaction regions with General Model Misspecification. In the left panel we fix the parameter $\hat{\alpha}$ to its rational value α, and let $\hat{\beta}$ vary. In the right panel $\hat{\beta}$ is kept fixed at β and $\hat{\alpha}$ varies.



B.2 General Expressions when $Z \neq 0$

This section generalizes the expressions of Section 3 when $Z \neq 0$. With a non-zero net supply of the asset, expressions are slightly more complicated but do not bring any new intuition. In the case of biases in learning from fundamentals, the equilibrium price becomes:

$$
\begin{aligned}
P = & \frac{\phi g(\tau_s + \tau_0)}{\tau_0 + \phi\tau_s + (1-\phi)\hat{\tau}_s} s + \frac{(1-\phi)\hat{g}(\hat{\tau}_s + \tau_0)}{\tau_0 + \phi\tau_s + (1-\phi)\hat{\tau}_s} \tilde{s} \\
& + \frac{\left(\phi(1-g)(\tau_s+\tau_0) + (1-\phi)(1-\hat{g})(\hat{\tau}_s+\tau_0)\right)\mu_0}{\tau_0 + \phi\tau_s + (1-\phi)\hat{\tau}_s} - \frac{\mathcal{A}Z}{\tau_0 + \phi\tau_s + (1-\phi)\hat{\tau}_s}
\end{aligned}
\tag{B.5}
$$

which can be rewritten:

$$
P = \hat{\alpha} s + \hat{\beta}\tilde{s} + (1 - \hat{\alpha} - \hat{\beta})\mu_0 - \hat{\delta} Z \tag{B.6}
$$

where, typically, $\hat{\delta} < \delta^{REE}$ when agents have over-reaction biases. The misspecified model agents learn from must now be expressed as:

$$
P = \tilde{\gamma}\tilde{s} + (1 - \tilde{\gamma})\mu_0 - \tilde{\delta} Z \tag{B.7}
$$

These slight changes have no impact on our propositions on aggregate demand elasticities or stability. The equilibrium is now given by:

$$P = \frac{\hat{\alpha}}{1 - \frac{\hat{\beta}}{\tilde{\gamma}}} s + \left(1 - \frac{\hat{\alpha}}{1 - \frac{\hat{\beta}}{\tilde{\gamma}}}\right) \mu_0 - \left(\hat{\delta} - \frac{\hat{\beta}}{\tilde{\gamma}} \frac{\tilde{\delta} - \hat{\delta}}{1 - \frac{\hat{\beta}}{\tilde{\gamma}}}\right) Z \tag{B.8}$$

so that there is now a constant term changing the level of prices independently of information, that depends on the sign of $\hat{\delta} - \tilde{\delta}$. All the comparative statics with respect to s are exactly the same as with $Z = 0$.

C Endogenous Information Acquisition and Stability

In our main framework, we emphasized that the equilibrium can be unstable when there are too few informed traders (Lemma 1). We also showed that even when the equilibrium is stable, mispricing can become arbitrarily large as we vary ϕ or τ_s/τ_0 to approach unstable regions.[2] The greater mispricing increases the profits of informed traders, therefore providing an incentive for endogenous information acquisition. This section considers endogenous information acquisition both on the intensive and on the extensive margin, and shows that it can contribute to ensuring the equilibrium is stable. That said, even with endogenous entry there are environments that lead the equilibrium to *approach* unstable regions, and our insights about amplification are robust to these extensions.

C.1 Endogenous Information Acquisition by Informed Investors

We normalize $\mu_0 = 0$ and $Z = 0$ for simplicity. The rest of the setup is unchanged, other than for the information structure. We still assume that there are ϕ informed traders and $1 - \phi$ uninformed traders, but now informed traders can decide the precision of the information they receive. To model endogenous information acquisition on the intensive margin, assume that there are an infinite number of infinitesimal signals in the background, denoted by s_i,

[2] We view this as a feature of our model: unstable regions in this case are just symptomatic of a very strong feedback effect between prices and beliefs, and this can lead prices and beliefs to become decoupled from fundamentals, as during the formation of bubbles. In Bastianello and Fontanier (2023) we exploit these insights to develop a model of bubbles and crashes, and offer a unifying framework of both normal time market dynamics (when shocks maintain the economy in a stable region) and of the formation of bubbles and crashes (when displacement shocks temporarily shift the economy in unstable regions).

such that if an investors acquires n signals, the precision of the information they receive is $n\tau_s$. Denote by $C(n)$ the cost of purchasing n signals, with $C'(n) > 0$. Informed investors can choose how many infinitesimal signals to receive, n.

The expected utility of an individual informed investor i choosing n signals, on the other hand, is:

$$\mathbb{E}_{i,n}[U] = \mathbb{E}_{i,n}\left[\frac{1}{2}\frac{(\mathbb{E}_{I,n}[v] - P)^2}{A\mathbb{V}_{I,n}[v]}\right] \tag{C.9}$$

where $\mathbb{E}_{i,n}[v]$ is the point estimate of the investor *after* she acquires n signals:

$$\mathbb{E}_{i,n}[v] = \frac{n\tau_s}{n\tau_s + \tau_0}\frac{1}{n}\int_0^n s_i di \tag{C.10}$$

and $\mathbb{V}_{i,n}[v] = (n\tau_s + \tau_0)^{-1}$. We then want to understand how investor i optimally chooses n, conditional on the rest of the market choosing precision N.

Specifically, let N be the quantity of signals acquired by every other informed investor, such that the "precision chosen by the market" is $N\tau_s$. To keep this extension as close as possible to our main framework, we assume that these information choices are public. As such, PET investors realize that informed traders are trading with precision $N\tau_s$. Given this, an individual informed investor understands that the price will be equal to the same PET price function we derived in the main text. For each signals realizations, the equilibrium price will thus be equal to:

$$P = \left(\frac{N\tau_s}{N\tau_s + \left(\frac{2\phi-1}{\phi^2}\right)\tau_0}\right)\frac{1}{N}\int_0^N s_i di = a(N)\frac{1}{N}\int_0^N s_i di \tag{C.11}$$

where in the second equality we define $a(N) \equiv \left(\frac{N\tau_s}{N\tau_s + \left(\frac{2\phi-1}{\phi^2}\right)\tau_0}\right)$ for simplicity.

To solve informed trader i's information acquisition problem, we first need to expand the expression for the expected square value of $(\mathbb{E}_{in,}[v] - P)$ for any (n, N) pair chosen by the individual investor and the market:

$$\mathbb{E}(\mathbb{E}_{i,n}[v] - P)^2 = \mathbb{E}\left(\frac{n\tau_s}{n\tau_s + \tau_0}\frac{1}{n}\int_0^n s_i di - a(N)\frac{1}{N}\int_0^N s_i di\right)^2 = \mathbb{E}\left(\frac{n\tau_s}{n\tau_s + \tau_0}\bar{s}_n - a(N)\bar{s}_N\right)^2 \tag{C.12}$$

where in the second equality we simply define $\bar{s}_n \equiv \frac{1}{n}\int_0^n s_i di$ and $\bar{s}_N \equiv \frac{1}{N}\int_0^N s_i di$ for conciseness. Simplifying further, we get:[3]

$$\mathbb{E}(\mathbb{E}_{I,n}[v]-P)^2 = \left(\frac{n\tau_s}{n\tau_s+\tau_0}\right)^2\left(\frac{n\tau_s+\tau_0}{n\tau_s\tau_0}\right) + a(N)^2\left(\frac{N\tau_s+\tau_0}{N\tau_s\tau_0}\right) - 2\frac{n\tau_s}{n\tau_s+\tau_0}a(N)\left(\frac{1}{\tau_0}+\frac{1}{max(n,N)\tau_s}\right) \quad \text{(C.13)}$$

Dividing this expression by $\text{Var}_{i,n}[v] = (n\tau_s+\tau_0)^{-1}$, and assuming that $n \geq N$ without loss of generality, we have that agent i's expected utility is proportional to the following:

$$2A\mathbb{E}_{i,n}[U] = \frac{n\tau_s}{\tau_0} + a(N)^2(n\tau_s+\tau_0)\left(\frac{N\tau_s+\tau_0}{N\tau_s\tau_0}\right) - 2n\tau_s a(N)\left(\frac{1}{\tau_0}+\frac{1}{n\tau_s}\right) \quad \text{(C.14)}$$

Taking the derivative of this expression, setting the marginal benefit of acquiring more information equal to the marginal cost, and notcingin that in equilibrium $n = N$, we find that the optimal amount of information N is pinned down by the following expression:

$$\frac{\tau_s}{\tau_0} + a(N)^2\tau_s\left(\frac{N\tau_s+\tau_0}{N\tau_s\tau_0}\right) - 2\tau_s a(N)\left(\frac{1}{\tau_0}\right) = 2AC'(N) \quad \text{(C.15)}$$

so that equation (C.15) implicitly defines the equilibrium N.

C.1.1 Equilibrium Solution

We numerically solve for the equilibrium N by solving equation C.15, using a constant marginal cost for additional information, i.e. a linear cost function C. Figure 4 shows that with no endogenous information acquisition (red line) the equilibrium becomes unstable when the fraction of informed traders is too low; instead, with endogenous information acquisition (blue line) the strength of the feedback effect always stays below 1 when agents

[3]Specifically, we use the fact that:

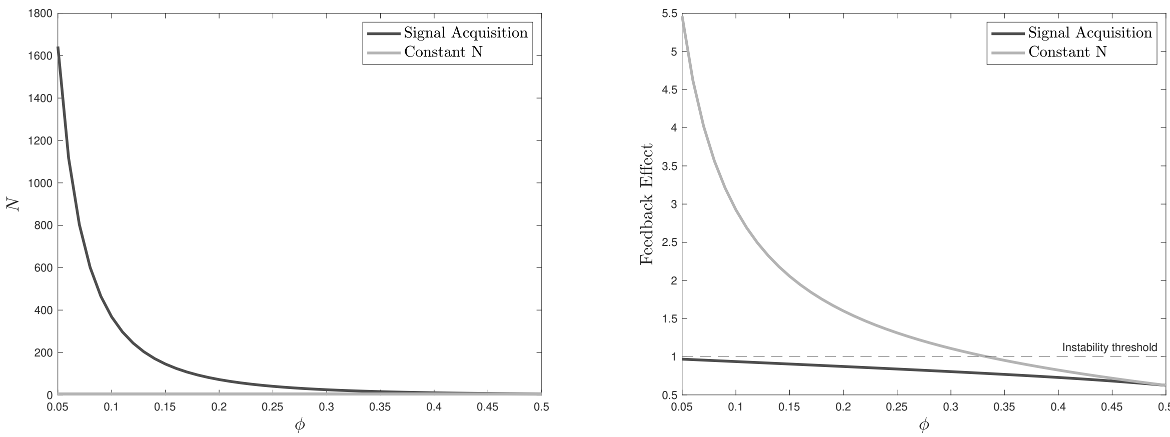
$$Var(\bar{s}_n) = \frac{1}{\tau_0} + \frac{1}{n\tau_s} = \frac{n\tau_s+\tau_0}{n\tau_s\tau_0}$$

$$Cov(\bar{s}_N, \bar{s}_n) = \frac{1}{nN}Cov\left(\int_0^N (v+\epsilon_i)di, \int_0^n (v+\epsilon_i)di\right) = \frac{1}{nN}Cov\left(Nv + \int_0^N \epsilon_i di, nv + \int_0^n \epsilon_i di\right)$$

$$= \frac{1}{\tau_0} + Cov\left(\int_0^{min(n,N)} \epsilon_i di, \int_0^{min(n,N)} \epsilon_i di\right) = \frac{1}{\tau_0} + \frac{min(n,N)}{nN\tau_s} = \frac{1}{\tau_0} + \frac{1}{max(n,N)\tau_s}$$

choose optimally how many signals to purchase, so that the equilibrium is always stable.

Figure 4: Intensive Margin in Information Choice and Stability. This figure shows the influence of endogenous information acquisition on stability. The left panel plots the number of signals acquired by informed investors when they face a constant marginal cost of acquiring supplementary signals (blue line): as the fraction of informed traders decreases, their benefits from acquiring information increase, leading them to acquire more signals. The red line keeps N constant, at the level chosen optimally by agents when $\phi = 0.5$. The right panel plots the equilibrium strength of the feedback effect with and without endogenous information acquisition. The dotted line shows when the feedback effect is above or below one, which is the threshold that determines whether the equilibrium is stable or unstable. With endogenous information acquisition, the equilibrium is always stable, even though it can still approach unstable regions.



Notice that, while the instability region disappears, the feedback effect still approaches 1 in the limit. This means that the insights we uncovered in the main framework about having an arbitrary amount of amplification are still present, albeit for lower levels of ϕ. In this sense, endogenous information acquisition on the intensive margin gives our equilibrium discipline by avoiding unstable equilibria, while retaining the validity of our results.

C.1.2 Incentives and Stability

What is the intuition for whether information choices prevent instability? From the previous derivation we know that expected utility with respect to n can be expressed as:

$$\frac{d}{dn} 2A\mathbb{E}_{i.n}[U] = \frac{\tau_s}{\tau_0} + a(N)^2 \tau_s \left(\frac{N\tau_s + \tau_0}{N\tau_s\tau_0} \right) - 2\tau_s a(N) \left(\frac{1}{\tau_0} \right) \tag{C.16}$$

When this expression is larger than $C'(n)$, individual informed investors have an incentive to purchase more signals. This expression is quadratic in $a(N)$, the price sensitivity coefficient expected by informed investors. It is then clear that, all else being equal, this expression goes to infinity when $a(N)$ goes to infinity: $\frac{d\mathbb{E}_{i,n}[U]}{dn} \xrightarrow[a(N)\to+\infty]{} +\infty$. Furthermore, we showed in our baseline framework that the price sensitivity coefficient was growing unboundedly large *precisely* when we approach the instability region. Intuitively, this means that when the coefficients of the model are such that we are close to being in an unstable region, informed investors have an incentive to purchase more information. This in itself creates a corrective force that pushes towards a stable equilibrium (as in Lemma 1).

This corrective force can also be understood by simply looking at the expected utility expression:

$$\mathbb{E}_I\left[\frac{1}{2}\frac{(\mathbb{E}_{i,n}[v]-P)^2}{A\text{Var}_I}\right] \tag{C.17}$$

We know that when approaching the unstable region, the equilibrium price goes towards $+\infty$ or $-\infty$ almost surely.[4] Informed agents thus expect to reap large profits on average, which increase their incentives to invest in higher precision, in order to trade more aggressively and take advantage of this predictable mispricing.

C.2 Entry of Informed Traders

We normalize $\mu_0 = 0$ and $Z = 0$ for simplicity. The rest of the setup is unchanged, other than for the information structure. We assume that there is a measure φ of uninformed (misspecified) traders, and N informed traders, where we are going to endogenize N. Informed traders receive a common signal $s|v \sim N(v, \tau_s^{-1})$. Uninformed traders do not observe the signal, but use the following misspecified mapping to extract information from prices:

$$P^{mis} = \tilde{\gamma}\tilde{s} \tag{C.18}$$

Price Function for a given N. The market clearing price function conditional on having N informed agents trading on s and φ uninformed agents trading on $\tilde{s}$ can be expressed as:

$$P = \frac{N\tau_s}{(N+\varphi)(\tau_s+\tau_0)}s + \frac{\varphi\tau_s}{(N+\varphi)(\tau_s+\tau_0)}\tilde{s} \tag{C.19}$$

[4] It only stays constant for the risk-adjusted neutral signal, a zero-probability event.

Substituting the signal inferred from (C.18) into (C.19), we find the following equilibrium price function:

$$P = \frac{N\tau_s}{(N+\varphi)(\tau_s+\tau_0) - \frac{\varphi\tau_s}{\tilde{\gamma}}} s \tag{C.20}$$

where the equilibrium is stable if the denominator of this expression is positive. Before going any further, notice that for the equilibrium to be stable, the denominator must be positive, and the number of informed traders must be high enough such that:[5]

$$N > \varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}} - 1\right) \tag{C.21}$$

In what follows we want to understand whether endogenous entry ensure that this condition is satisfied.

Endogenizing N. The ex-ante expected utility of entering for a trader who thinks there are N informed traders and φ uninformed traders is given by the following expression:

$$\mathbb{E}_I\left[\frac{1}{2}\frac{(\mathbb{E}_I[v]-P)^2}{A\mathrm{Var}_I}\right] = \frac{\tau_s+\tau_0}{2A\tau_0}\left(\frac{\tau_s}{\tau_s+\tau_0} - \frac{N\tau_s}{(N+\varphi)(\tau_s+\tau_0)-\frac{\varphi\tau_s}{\tilde{\gamma}}}\right)^2 \tag{C.22}$$

Moreover, assume that it costs C for a trader to enter. Traders will enter(exit) as long as the expected benefit from entering is greater(lower) than the cost of entering. In equilibrium, it must be that the expected benefit of entering is equal to the cost of entering. Setting (C.22) equal to C, and rearranging yields:

$$\frac{\tau_s+\tau_0}{2A\tau_0}\left(\frac{\varphi\tau_s}{\tau_s+\tau_0}\right)^2\left(\frac{\tau_s+\tau_0-\frac{\tau_s}{\tilde{\gamma}}}{(N+\varphi)(\tau_s+\tau_0)-\frac{\varphi\tau_s}{\tilde{\gamma}}}\right)^2 = C \tag{C.23}$$

The expression that appears in the denominator in the last fraction on the left hand side is exactly what determines stability. Denote the denominator of that fraction by $D \equiv (N+\varphi)(\tau_s+\tau_0) - \frac{\varphi\tau_s}{\tilde{\gamma}}$, such that we can re-write the equilibrium condition as:

[5] Notice that the condition for stability is the same as the one we obtained in the baseline model: $\frac{\partial P^{Mis}(\tilde{s})}{\partial \tilde{s}} > \frac{\partial P^{True}(\tilde{s};s)}{\partial \tilde{s}} \iff \tilde{\gamma} > \frac{\varphi\tau_s}{(N+\varphi)(\tau_s+\tau_0)} \iff (N+\varphi)(\tau_s+\tau_0) - \frac{\varphi\tau_s}{\tilde{\gamma}} > 0$.

$$\frac{\tau_s+\tau_0}{2A\tau_0}\left(\frac{\varphi\tau_s}{\tau_s+\tau_0}\right)^2\left(\frac{\tau_s+\tau_0-\frac{\tau_s}{\tilde{\gamma}}}{D}\right)^2=C$$

$$\implies D=\pm\sqrt{\frac{\tau_s+\tau_0}{2A\tau_0 C}}\left(\frac{\varphi\tau_s}{\tau_s+\tau_0}\right)\left(\tau_s+\tau_0-\frac{\tau_s}{\tilde{\gamma}}\right) \quad \text{(C.24)}$$

This formulation makes clear that, for any set of parameters, this equilibrium condition has two solutions for D: one positive and one negative. This means that there are two entry equilibria: one where the resulting equilibrium is stable, and one where the resulting equilibrium is unstable. However, the constraint that $N > 0$ means that some of these solutions are ruled out. In what follows, we show this explicitly by solving for N.

Using the definition of D in (C.24), and rearranging gives the following solution for N:

$$N=\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right)\left(1\pm\frac{\tau_s}{\sqrt{2A\tau_0(\tau_s+\tau_0)C}}\right)>0 \quad \text{(C.25)}$$

where the last inequality just highlights that we need the number of informed traders in the market to be positive.[6] We can then consider two different cases in turn, which we also discuss in Figure 5.

Case 1. $\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right)<0$. When this is the case, the constraint that $N>0$ implies that a solution only exists when $C<\frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$, and that as long as the entry cost is low enough that this condition is satisfied, then the equilibrium is unique and stable:

$$N_{eq}=\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right)\left(1-\frac{\tau_s}{\sqrt{2A\tau_0(\tau_s+\tau_0)C}}\right)>0>\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right) \quad \text{(C.26)}$$

where the last inequality highlights that the condition for stability in (C.21) is indeed satisfied. This is illustrated in the left panel of Figure 5. Finally, notice how as C increases towards $\frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$, the denominator D approaches 1, meaning that we can still approach unstable regions, even if the equilibrium is stable. Intuitively, as entry costs increase, it becomes more costly for informed agents to earn profits, and the lack of their corrective force contributes to a stronger feedback effect.

Case 2. $\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right)>0$. When this is the case, there can be multiple equilibria for

[6] Remember that the number of informed traders is common knowledge. If there were no informed traders, then there would be no information in prices, and the inference problem would be meaningless.

$N > 0$, depending on the value of C, as shown in the right panel of Figure 5. If $C < \frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$ is low enough, then there is a unique equilibrium that is stable:

$$N_1 = \varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}} - 1\right)\left(1 + \frac{\tau_s}{\sqrt{2A\tau_0(\tau_s+\tau_0)C}}\right) > \varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}} - 1\right) > 0 \qquad \text{(C.27)}$$

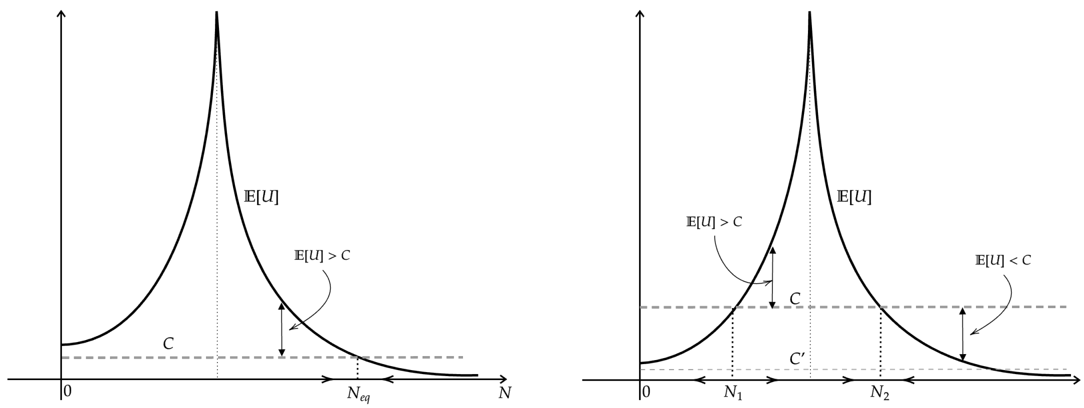
If instead $C > \frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$ then there is also a second equilibrium, which instead is unstable:

$$0 < N_2 = \varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}} - 1\right)\left(1 - \frac{\tau_s}{\sqrt{2A\tau_0(\tau_s+\tau_0)C}}\right) < \varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}} - 1\right) \qquad \text{(C.28)}$$

Notice however, that this unstable equilibrium is also unstable on the entry margin, meaning that if N deviates from N_1, then further entry/exit will not converge to back to N_1. To see why this is the case, Figure 5 shows that if we perturb the number of informed traders to be just above N_1, then the expected payoff of entering is greater than the cost, meaning that more informed traders would enter (moving away from N_1, and towards the stable equilibrium N_2); when instead the number of informed traders is perturbed to be just below N_1, then the expected payoff of entering is lower than the cost, meaning that informed traders would exit (again moving away from N_1). Therefore unless we start from the case with N_1 informed traders, endogenous entry would never return this equilibrium, making the stable equilibrium much more appealing.

We can summarize our results as follows: as long as the cost of entering is low enough $C < \frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$, the equilibrium is always unique and stable. If the cost of entering is too high $C > \frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$, then there is either no equilibrium, or there are multiple equilibria, where the second equilibrium is unstable both on the entry margin, and in terms of outcomes. Interestingly, even when the equilibrium is stable, we can still approach unstable regions, meaning that our results of increasing and arbitrarily large deviations from rationality as we approach unstable regions still hold.

Figure 5: Equilibrium Entry and Stability. This figure shows the equilibrium determination of N for a given cost parameter C. The left panel corresponds to Case 1: when $\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right)<0$, there is a unique and stable solution in N as long as $C<\frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$. The right panel corresponds to our Case 2: when $\varphi\left(\frac{\tau_s}{\tau_s+\tau_0}\frac{1}{\tilde{\gamma}}-1\right)>0$ there is a unique stable equilibrium if $C<\frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$, while there is an additional unstable equilibrium if $C>\frac{\tau_s^2}{2A\tau_0(\tau_s+\tau_0)}$. The arrows on the x-axis also illustrates how N_{eq} and N_2 are always stable on the entry margin, while N_1 is unstable on the entry margin when it exists.



D K-Level Thinking

PET can be understood as 2− level thinking, while the fully cursed equilibrium (CE) is 1− level thinking. In the CE, agents do not infer information and trade on their private information alone, while in PET everyone believes that others are 1− level thinkers. The natural next step is to consider what happens as we allow for rationality of higher order beliefs, whereby a $K-$level thinker believes that all other agents are $(K-1)-$level thinkers.[7] For example, a 3−level thinker believes that all other agents think they are the only ones extracting information from prices.

D.1 Solving for K-Level Thinking

We solve for $K-$level thinking recursively by following the same steps outlined in Section 2.2. First, in extracting information from prices, all U agents *believe* that all other U agents are $(K-1)-$level thinkers, and that the price they observe is generated by the $(K-1)-$level

[7] Many papers have tested $K-$level thinking and estimated low levels of K, between 0 and 3 (Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006), Crawford et al. (2013))

thinking equilibrium price function:

$$P^{Mis}(\tilde{s}) = \tilde{\gamma}_{K-1}\tilde{s} + (1-\tilde{\gamma}_{K-1})(\mu_0 - AZ\tau_0^{-1}) \tag{D.29}$$

where $\tilde{\gamma}_{K-1}$ is the sensitivity of the price to the true signal in the $(K-1)-$level thinking equilibrium, and $\tilde{s}$ here denotes a $K-$level thinker's belief about s. Let $1-$level thinking to be the cursed equilibrium (CE), where all traders trade on their private information. This leads to $\tilde{\gamma}_1 \equiv \frac{\phi\tau_s}{\phi\tau_s+\tau_0}$.

Second, in *reality*, all I agents trade on the true signal s, and all U agents are $K-$level thinkers and trade on the signal $\tilde{s}$, which they extract from prices using the mapping in (D.29). Given these beliefs, the true market clearing condition leads to:

$$P^{True}(s,\tilde{s}) = \alpha s + \beta\tilde{s} + (1-\alpha-\beta)(\mu_0 - AZ\tau_0^{-1}) \tag{D.30}$$

where $\alpha \equiv \frac{\phi\tau_s}{\tau_s+\tau_0}$, $\beta \equiv \frac{(1-\phi)\tau_s}{\tau_s+\tau_0}$, and are constant and independent of K.

Finally, in *equilibrium*, agents' (misspecified) beliefs must be consistent with the price they observe. Solving for the $(P_K, \tilde{s}_K)-$pair which satisfy (D.29) and (D.30) jointly, provides us with a recursive solution for $K > 1$, starting from the CE equilibrium with $\tilde{\gamma}_1 \equiv \frac{\phi\tau_s}{\phi\tau_s+\tau_0}$:

$$P_K = \tilde{\gamma}_K s + (1-\tilde{\gamma}_K)(\mu_0 - AZ\tau_s^{-1}) \tag{D.31}$$

$$\tilde{s}_K = \frac{\tilde{\gamma}_K}{\tilde{\gamma}_{K-1}} s + \left(1 - \frac{\tilde{\gamma}_K}{\tilde{\gamma}_{K-1}}\right)\left(\mu_0 - AZ\tau_0^{-1}\right) \tag{D.32}$$

where:

$$\tilde{\gamma}_K = \frac{\alpha}{1 - \frac{\beta}{\tilde{\gamma}_{K-1}}} \tag{D.33}$$

Solving (D.33) forwards yields the following equilibrium sensitivity for $K > 1$:

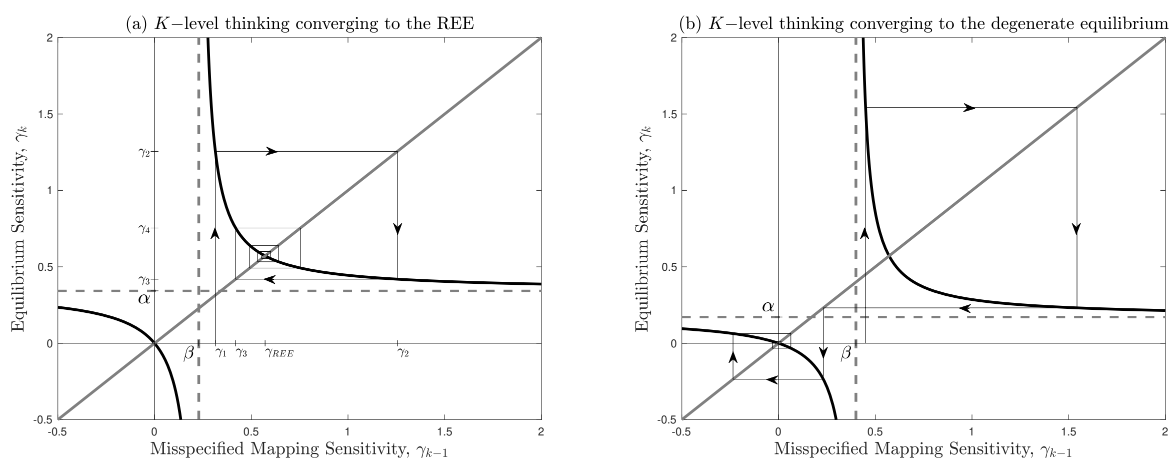
$$\tilde{\gamma}_K = \frac{\alpha}{1 + \sum_{j=1}^{K-2}\left(-\frac{\beta}{\alpha}\right)^j - \left(-\frac{\beta}{\alpha}\right)^{K-2}\frac{\beta}{\tilde{\gamma}_1}} = \frac{\tau_s}{\tau_s + \tau_0 - \left(-\frac{1-\phi}{\phi}\right)^K \tau_0} \tag{D.34}$$

where the second equality uses the expressions for α and β.

D.2 Properties of K-level Thinking

Let us start by studying the properties of the recursive relationship in (D.33), as this fully characterizes $K-$level thinking equilibrium outcomes in (D.31) and (D.32). Figure 6 plots this relationship in bold, together with the 45° line. The intersections of these two lines correspond to two fixed points such that $\tilde{\gamma}_{K-1} = \tilde{\gamma}_K$: $\tilde{\gamma}_K = \alpha + \beta > 0$ is the REE, and $\tilde{\gamma}_K = 0$ is a degenerate fixed point where the price is unresponsive to the signal. The arrows then show the recursive evolution of $\tilde{\gamma}_K$ as we allow for rationality of higher order beliefs. Figure 6 makes clear three properties of the $K-$level thinking equilibrium, two of which generalize the intuitions on stability and over-reaction we uncovered in Section 2, and the third relates to convergence to the REE as we allow for rationality of higher order beliefs. We discuss them in turn.

Figure 6: Convergence of $K-$level thinking. This Figure plots the recursive relationship in (D.33) in bold: the $x-$axis tracks the sensitivity of the misspecified mapping of a level K thinker, and the $y-$axis tracks the corresponding sensitivity of the $K-$level thinking equilibrium price. The diagonal line is the 45° line. The intersection of these two lines correspond to two fixed points such that $\tilde{\gamma}_{K-1} = \tilde{\gamma}_K$: $\tilde{\gamma}_K = \alpha+\beta > 0$ is the REE, and $\tilde{\gamma}_K = 0$ is a degenerate fixed point. The arrows show the recursive evolution of $\tilde{\gamma}_K$ as we allow for rationality of higher order beliefs, starting from $\tilde{\gamma}_1$. The left panel depicts a case where $\alpha > \beta$, and the left panel shows a case for $\alpha < \beta$.



Proposition 12 (Stability and $K-$Level Thinking)**.** *The $K-$level thinking equilibrium is stable if and only if* $\beta/\tilde{\gamma}_{K-1} < 1$. *Otherwise, the equilibrium is unstable.*

Proof. The aggregate excess demand function takes the following form:

$$X_{TOT}^K - Z = \phi\left(\frac{\frac{\tau_s}{\tau_s+\tau_0}s + \frac{\tau_0}{\tau_s+\tau_0}\mu_0 - P_K}{A(\tau_s+\tau_0)^{-1}}\right) + (1-\phi)\left(\frac{\frac{\tau_s}{\tau_s+\tau_0}\tilde{s}_K + \frac{\tau_0}{\tau_s+\tau_0}\mu_0 - P_K}{A(\tau_s+\tau_0)^{-1}}\right) \quad \text{(D.35)}$$

Moreover, from (D.29), we know that $K-$level uninformed agents use the following mapping to extract information from prices: $\tilde{s}_K = \frac{1}{\tilde{\gamma}_{K-1}} P_K - \frac{1-\tilde{\gamma}_{K-1}}{\tilde{\gamma}_K}(\mu_0 - AZ\tau_0^{-1})$. Substituting this into (D.35), and rearranging, we get:

$$X_{TOT}^K - Z = \frac{1}{A(\tau_s + \tau_0)^{-1}} \left(\frac{(1-\phi)\tau_s}{\tau_s + \tau_0} \frac{1}{\tilde{\gamma}_{K-1}} - 1 \right) P_K + constants \tag{D.36}$$

and since $\frac{(1-\phi)\tau_s}{\tau_s+\tau_0} = \beta$, we can rewrite this as:

$$X_{TOT}^K - Z = \underbrace{\frac{1}{A(\tau_s + \tau_0)^{-1}} \left(\frac{\beta}{\tilde{\gamma}_{K-1}} - 1 \right)}_{\text{slope of excess demand function}} P_K + constants \tag{D.37}$$

Therefore, the $K-$level thinking equilibrium is stable if and only if the slope of this excess demand function is negative, which is equivalent to:

$$\left(\frac{\beta}{\tilde{\gamma}_{K-1}} - 1 \right) < 0 \iff \frac{\beta}{\tilde{\gamma}_{K-1}} < 1 \tag{D.38}$$

Since $\beta > 0$, this condition is satisfied either if $\tilde{\gamma}_{K-1} < 0$, or if $0 < \beta < \tilde{\gamma}_{K-1}$. Otherwise, the excess demand function is upward sloping, and the equilibrium unstable. □

This result generalizes our findings in Proposition 1. Intuitively, when $\tilde{\gamma}_{K-1} > 0$ and $\beta/\tilde{\gamma}_{K-1} > 0$, the informational role of prices introduces strategic *complementarities* which work against the strategic subsitutabilities from the scarcity role of prices, and push towards an upward sloping aggregate excess demand function. When this is the case, $\beta/\tilde{\gamma}_{K-1} < 1$ simply ensures that the scarcity role dominates over the informational role, so that the demand function is downward sloping, and the equilibrium is stable. On the other hand, notice that with $K-$level thinking we may now also encounter cases when $\tilde{\gamma}_{K-1} < 0$, according to which the informational role of prices introduces strategic *substitutabilities*. When this is the case, there is no tension between the two roles of prices: instead, the informational role reinforces the downward sloping nature of the demand function. Not only does this ensure stability, but it also *dampens* the general equilibrium adjustment, thus leading to under-reaction relative to the REE, as shown in Figure 6.[8,9]

Next, we turn to the properties of $K-$level thinking when the equilibrium is stable.

Proposition 13 ($K-$level Thinking Over-/Under-reaction)**.** *When the equilibrium is stable, $K-$level thinking outcomes exhibit over-reaction relative to the REE if and only if $\tilde{\gamma}_{K-1} < \tilde{\gamma}_{REE}$. Otherwise, they exhibit under-reaction.*

Proof. The equilibrium sensitivity of the $K-$level thinking equilibrium price to the true signal is given in equation (D.33):

$$\tilde{\gamma}_K = \frac{\alpha}{1 - \frac{\beta}{\tilde{\gamma}_{K-1}}} \tag{D.39}$$

We also know that the $K-$level thinking equilibrium is stable is equivalent to $\tilde{\gamma}_K > 0$. Since $\alpha, \beta > 0$, this is equivalent to $\tilde{\gamma}_{K-1} > \beta$. The REE sensitivity is given by:

$$\gamma_{REE} = \alpha + \beta. \tag{D.40}$$

By definition, the $K-$level thinking equilibrium exhibits over-reaction if $\tilde{\gamma}_K > \gamma_{REE}$:

$$\frac{\alpha}{1 - \frac{\beta}{\tilde{\gamma}_{K-1}}} > \alpha + \beta \iff \tilde{\gamma}_{K-1} < \alpha + \beta = \gamma_{REE} \tag{D.41}$$

where the last inequality uses the fact that the equilibrium is stable. Therefore, for a stable equilibrium to exhibit over-reaction, we need: $\beta < \tilde{\gamma}_{K-1} < \tilde{\gamma}_K$. □

This result is clear in Figure 6: only when $\beta < \tilde{\gamma}_{K-1} < \gamma_{REE}$ is the $K-$level thinking equilibrium stable and $\tilde{\gamma}_K > \gamma_{REE} = \alpha + \beta$.

Intuitively, focusing on stable outcomes, when $\tilde{\gamma}_{K-1} < \gamma_{REE}$, U agents think that all other agents are, on average, underreacting, and that the equilibrium price is less responsive

[8]The results from Proposition 12 can be seen in Figure 6 by noticing that $\tilde{\gamma}_K > 0$ if and only if $\tilde{\gamma}_{K-1} > \beta$ or $\tilde{\gamma}_{K-1} < 0$. Moreover, when $\tilde{\gamma}_{K-1} < 0$, $\tilde{\gamma}_K < \tilde{\gamma}_{REE}$ and the equilibrium exhibits under-reaction.

[9]The fact that $K-$level thinking can either dampen or amplify GE effects is consistent with Angeletos and Lian (2017), who study a complete information setup where agents have limited capacity to think through the response of others to a known shock. A different approach to studying wedges between partial and general equilibrium effects is in Angeletos and Lian (2018), Angeletos and Sastry (2020) and Angeletos and Huo (2021). They maintain common knowledge of rationality but add higher-order uncertainty: agents engage in a beauty contest style of thinking to forecast other agents' beliefs and responses to shocks, but there is no inference from endogenous outcomes.

to signals than it really is. Therefore, they attribute any price change they observe to more extreme news than in reality. Since all U agents behave in the same way, this translates into aggregate level over-reaction, just as in PET. Conversely, when $\tilde{\gamma}_{K-1} > \gamma_{REE}$, U agents think that other agents are, on average, overreacting, and their combined response leads to under-reaction in equilibrium. Therefore, in the specific case we consider (with level-1 being the CE), $K-$level thinking alternates between under-reaction when K is odd (and U agents think others are overreacting) and over-reaction when K is even (and U agents think others are underreacting). Section 3 makes this result about individual level inference more general and shows that it is independent of K.

We end this section by considering the conditions for $K-$level thinking to converge to the REE, as we allow for rationality of higher order beliefs.

Proposition 14 (Convergence to the REE)**.** *$K-$level thinking converges to the REE as we allow for rationality of higher orders beliefs if and only if $\alpha > \beta$. Otherwise, it converges to the degenerate fixed point with $\tilde{\gamma}_K = 0$.*

Proof. We start with the recursive formulation of the equilibrium sensitivity (D.33):

$$\tilde{\gamma}_K = \frac{\alpha}{1 - \frac{\beta}{\tilde{\gamma}_{K-1}}} \tag{D.42}$$

Solving for the fixed point of this equation, such that $\tilde{\gamma}_K = \tilde{\gamma}_{K-1}$, yields two solutions, $\tilde{\gamma}_K = \alpha+\beta = \gamma_{REE}$ and $\tilde{\gamma}_K = 0$ (these are also shown in Figure 6). Define the convergence function as $f(\tilde{\gamma}_{K-1}) = \frac{\alpha}{1-\frac{\beta}{\tilde{\gamma}_{K-1}}}$. The REE fixed point is attractive when the following condition is met:

$$|f'(\gamma_{REE})| < 1 \tag{D.43}$$

Straightforward algebra yields that this is verified when:

$$\frac{\alpha\beta}{(\gamma_{REE} - \beta)^2} < 1 \iff \frac{\alpha\beta}{(\alpha + \beta - \beta)^2} < 1 \iff \alpha > \beta \tag{D.44}$$

Similarly, the attractive fixed point is the degenerate one if:

$$|f'(0)| < 1 \tag{D.45}$$

which requires:

$$\frac{\alpha\beta}{(0-\beta)^2} < 1 \iff \alpha < \beta. \tag{D.46}$$

Using the values of α and β, we also notice that, in our setup, (D.44) is satisfied if and only if $\phi > 1/2$. □

This result is intuitive in light of equation (D.34), where $\lim_{K\to\infty}\tilde{\gamma}_K = \alpha + \beta = \gamma_{REE}$ if $\alpha > \beta$ and $\lim_{K\to\infty}\tilde{\gamma}_K = 0$ if $\alpha < \beta$. Moreover, notice how convergence is independent of the initial size of the bias, $\tilde{\gamma}_1$. Instead, for $K-$level thinking outcomes to converge to the REE, we simply need the equilibrium influence on prices of I agents (α) to be greater than the influence on prices of U agents (β).[10] Figure 6 depicts examples of convergence to the REE and degenerate fixed points, in the left and right panels, respectively. The right panel of Figure 6 also show that when $\alpha < \beta$ there always exists a K large enough such that the $K-$level thinking equilibrium becomes unstable.

In our framework the condition for convergence reduces to $\phi > 1/2$. This suggests that a market populated by more U agents than I agents may exhibit smaller deviations from the REE when agents have *lower* levels of K. If one were to interpret agents with higher levels of K as being more sophisticated than their low K counterparts, the above result implies that greater levels of sophistication may contribute to greater (rather than lower) mispricing and to instability. Only when $\alpha > \beta$ and the market is populated by more I than U agents do greater levels of sophistication bring us closer to the REE. We summarize these results in Corollary 4.

Corollary 4 (Deviations from REE and Rationality of Higher Order Beliefs)**.** *When $\alpha > \beta$, the deviations of $K-$level thinking equilibrium outcomes from the REE are decreasing in K. Conversely, when $\alpha < \beta$, the deviations of $K-$level thinking equilibrium outcomes from the REE are increasing in K for stable equilibria, and there exists a K large enough such that the equilibrium becomes unstable.*

Empirically, this suggests that we are likely to observe greater mispricing when the market

[10] When this is the case, the size of the bias shrinks as it gets compounded with K. Conversely, when $\alpha < \beta$, the extent of over-reaction in PET (2−level) is greater in absolute value than the extent of under-reaction in CE (1−level). The mapping used by 3−level agents then deviates more from the REE than the one used by 2−level thinkers, thus translating into even greater equilibrium deviations from the REE outcomes, and to even more misspecified mappings for higher level thinkers.

is populated by a large fraction of U agents with high levels of rational higher order beliefs. In these scenarios, agents are more likely to chase mispricing rather than information. More generally, our results illustrate the fragility of the REE assumption in settings where agents are learning from endogenous outcomes.

E Heterogenous Agents

We have so far considered the case where all uninformed agents extract information from prices using the same misspecified model of the world, be it PET or any general form of misspecified model. The empirical literature however document a large amount of heterogeneity among agents' actions and beliefs (see for example Giglio et al. 2020). The tractability of our model allows us to consider the interaction of heterogeneous agents, who differ in their level of sophistication in extracting information from prices.[11]

E.1 Misspecified Models

Let there be a fraction of informed agents, and a fraction of uninformed agents. Moreover, let there be N different types of uninformed agents, each using a different misspecified model of the world to extract information from prices.

Misspecified model of the world for each agent $n = 1, 2, \ldots, N$. To solve the model, we need to specify the mapping used by each type of agent to extract information from prices. In this section we allow agents to have general *linear* mappings.[12,13] We denote these

[11]Most papers studying model misspecification assume that all individuals share the same bias. Notable exceptions include Gagnon-Bartsch (2016); Frick et al. (2019); and Bohren and Hauser (2021), who characterizes long-run beliefs in general settings with model heterogeneity.

[12]As shown in the main text, the linearity assumption is unrestrictive for CE, PET, REE and $K-$level thinking, and in section 3 we also discussed how this assumption nests many types of behavioral biases studied in the literature.

[13]In a setup where there are heterogeneous agents extracting information from prices, this linearity assumption simply requires that each agent n thinks that the (misspecified) mappings used by other agents are linear. When this is the case, agent n believes that other agents' (wrong) extracted signals are linear functions of the true signal. Therefore, when each agent n aggregates his/her perception of all other agents' linear asset demands, the resulting market clearing price function which determines his/her (misspecified) mapping is also linear. In this case, the details of what each agent n thinks about other agent's extracted signals determines $\tilde{\gamma}_n$, but not the functional form of their (misspecified) mapping.

models as:[14]

$$P^{Mis_n} = \tilde{\gamma}_n \tilde{s}_n + (1 - \tilde{\gamma}_n)\zeta_1 n \tag{E.47}$$

So that inverting prices for each type of agent gives the following extracted signal:

$$\tilde{s}_n = \frac{1}{\tilde{\gamma}_n} P^{Mis_n} - \left(\frac{1 - \tilde{\gamma}_n}{\tilde{\gamma}_n}\right)\zeta_n \tag{E.48}$$

True Model of the World. Given these N types of agents, the true price function takes the following form:

$$P^{True} = \alpha s + \sum_{n=1}^{N} \beta_n \tilde{s}_n + \delta, \tag{E.49}$$

where α, β_n for $n \in \{1, \ldots, N\}$ and δ are constant coefficients that depend on the composition of agents and fundamental parameters of the economy.

E.2 Equilibrium With Heterogeneous Agents

The equilibrium price and extracted signals again simply require that: $P = P^{Mis_1} = P^{Mis_2} = ... = P^{Mis_N}$. This yields the equilibrium price P:

$$P = \frac{\alpha}{1 - \sum_{n=1}^{N} \frac{\beta_n}{\tilde{\gamma}_n}} s + \frac{\delta - \sum_{n=1}^{N} \frac{\beta_n(1-\tilde{\gamma}_n)}{\tilde{\gamma}_n}\zeta_n}{1 - \sum_{n=1}^{N} \frac{\beta_n}{\tilde{\gamma}_n}} \tag{E.50}$$

and the extracted signals for each type:

$$\tilde{s}_n = \frac{\frac{\alpha}{\tilde{\gamma}_n}}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\tilde{\gamma}_m}} s + \frac{1}{\tilde{\gamma}_n}\left(\frac{\delta - \sum_{m=1}^{N} \frac{\beta_m(1-\tilde{\gamma}_m)}{\tilde{\gamma}_m}\zeta_m}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\tilde{\gamma}_m}} - (1 - \tilde{\gamma}_n)\zeta_n\right) \tag{E.51}$$

Notice that the resulting equilibrium is strikingly similar to the basic framework we explored in the main text.[15] In what follows, we show how stability and over-/under-reaction generalize to this setup with heterogeneous agent.

[14] Notice that ζ_i could simply be equal to $(\mu_0 - AZ\tau_0^{-1})$ as in the analysis in the main text. In this section we leave it general as, for example, agents may believe that other agents have different priors.

[15] To add generality we did not take a stance on the coefficients ζ_n and δ of the price functions. It is easy to verify that if we assume that $\zeta_i = \mu_0 - AZ\tau_0^{-1}$ for all i, and $\delta = \left(1 - \alpha - \sum_{n=1}^{N} \beta_n\right)(\mu_0 - AZ\tau_0^{-1})$, we are back to the case we described in details in Section 3.

Proposition 15 (Stability with Heterogeneous Agents)**.** *In an economy with N different types of agents who extract information from prices using the linear mappings described in* (E.49)*, the equilibrium is stable if and only if:*

$$\sum_{m=1}^{N} \left(\frac{\beta_m}{\tilde{\gamma}_m} \right) < 1 \tag{E.52}$$

This follows immediately from noticing that, as before aggregade demand is downward-sloping if and only if the cofficient on the signal in equation (E.50) is positive, and the assumption that $\alpha > 0$.

Notice that while (E.52) may seem more restrictive than the equivalent condition in Proposition 1, this is not necessarily the case. To see why, recall from (E.49) that β_n captures the relative influence on equilibrium prices of agent n's beliefs: the greater N is, the smaller β_n is, on average. Similarly, from (E.49), $\tilde{\gamma}_n$ is agent n's perceived sensitivity of the equilibrium price to new information: the greater the number of agents extracting information from prices, the more likely it is that uninformed agents think the equilibrium price is more sensitive to new information.

Next, we turn to characterize over-and under-reaction at the individual and aggregate levels in the following two propositions.

Proposition 16 (Over- and Under-Reaction at the Individual Level)**.** *When the equilibrium is stable, an agent of type n overreacts to changes in the unobserved signal s if and only if:*

$$\tilde{\gamma}_n < \frac{\alpha}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\tilde{\gamma}_m}} = \frac{\partial P}{\partial s} \tag{E.53}$$

and underreacts when the inequality is reversed.

This proposition is immediate from inspecting Equation (E.51)

This proposition shows that to determine an individual's level of over-/under-reaction, it is enough to compare the sensitivity of their mapping to a single aggregate statistic. Intuitively, individual agents extract the correct signal from prices if they use the true sensitivity of the equilibrium price to new information in their mapping. Agents who mistakingly think that the sensitivity is lower than in reality attribute a given price change to a more extreme signal, and overreact. Conversely, if agents think the sensitivity is greater than in reality they underreact.

When there is a single type of uninformed agent, this condition boils down to $\tilde{\gamma}_n < \alpha + \beta = \gamma_{REE}$, which is consistent with our earlier result in Proposition 5. However, whenever there are more than one type of agents, Proposition 16 shows that the correct benchmark to assess individual level over-reaction is the true equilibrium mapping, and not the REE one.

We now turn to the behavior of aggregate prices.

Proposition 17 (Over- and Under-Reaction at the Aggregate Level)**.** *The aggregate price function overreacts to changes in the signal s relative to the rational counterfactual if and only if:*

$$\frac{\alpha}{1 - \sum_{m=1}^{N} \frac{\beta_m}{\tilde{\gamma}_m}} > \alpha + \sum_{m=1}^{N} \beta_m \iff \sum_{n=1}^{N} \left(\frac{\beta_n}{\sum_{m=1}^{N} \beta_m} \frac{1}{\tilde{\gamma}_n} \right) > \frac{1}{\gamma_{REE}} \tag{E.54}$$

and it underreacts when the inequality is reversed.

This proposition follows from inspecting $\partial P / \partial s$ from equation (E.50), and using the fact that the rational counterfactual is such that $\tilde{\gamma}_n = \alpha + \sum_{m=1}^{N} \beta_m$ for all n.

The left-hand side of (E.54) is the weighted average of $1/\tilde{\gamma}_n$ across agents, where the weights are given by the relative influence that each agent has on equilibrium prices, $\beta_n / \sum_{m=1}^{N} \beta_m$. If we call this weighted average $1/\bar{\tilde{\gamma}}$, then the above expression implies that equilibrium prices exhibit over-reaction relative to the REE if and only if $\bar{\tilde{\gamma}} < \gamma_{REE}$. This is reminiscent of our earlier result in Proposition 5, which now generalizes to a setup with heterogeneous agents: if, on average, agents' sensitivity is lower than γ_{REE}, then, on average, they extract signals which are more extreme than in reality, leading to over-reaction in equilibrium outcomes.

Moreover, the first inequality in (E.54) also makes clear that when the equilibrium is stable the extent of over-reaction is increasing in the strength of the aggregate feedback effect, which can simply be represented as $\sum_{n=1}^{N} \frac{\beta_n}{\tilde{\gamma}_n}$.

F Partially Revealing Prices

This section generalizes PET to environments where prices are only partially revealing. This serves two purposes. First, it verifies that the insights we uncovered are robust to small variations in the information structure. Second, it allows us to expand our setup to a symmetric case where all agents receive a private signal, as shown in Appendix G.

To do so, we model the supply of the risky asset as being stochastic, for example because of the presence of noise traders. Therefore, the supply shock z is now drawn from a normal distribution, $z \sim N(\bar{z}, \tau_z^{-1})$. All other assumptions are the same as in our baseline framework.

To construct their demand functions, informed traders need to compute the distribution of $v|s$ (using simple Bayesian updating as we did throughout the main text), while uninformed traders need to compute the distribution of $(v, s, z)|P$. Moreover, uninformed traders still think that they lived in a cursed world. Using the same notation as in Section 3, we can write uninformed agents' misspecified model as:

$$P^{Mis} = \tilde{\gamma}\tilde{s} + (1-\tilde{\gamma})(\mu_0 - Az\tau_0^{-1}), \tag{F.1}$$

Uninformed traders cannot observe z, but they can use standard Bayesian updating to extract the maximum-likelihood signal. Following Diamond and Verrecchia (1981), we first write down the distribution of the information structure of the uninformed agents, $(v, s, z, P^{Mis}) \sim N(m, M)$, which is multivariate normal with mean m and variance-covariance matrix M, where:[16]

$$m = (\mu_0, \mu_0, \bar{z}, \mu_0 - (1-\tilde{\gamma})A\bar{z}\tau_0^{-1})^T + \tag{F.2}$$

$$M = \begin{pmatrix} \tau_0^{-1} & \tau_0^{-1} & 0 & \tilde{\gamma}\tau_0^{-1} \\ \tau_0^{-1} & \tau_0^{-1}+\tau_s^{-1} & 0 & \tilde{\gamma}(\tau_0^{-1}+\tau_s^{-1}) \\ 0 & 0 & \tau_z^{-1} & (1-\tilde{\gamma})\tau_z^{-1} \\ \tilde{\gamma}\tau_0^{-1} & \tilde{\gamma}(\tau_0^{-1}+\tau_s^{-1}) & (1-\tilde{\gamma})\tau_z^{-1} & \tilde{\gamma}^2(\tau_0^{-1}+\tau_s^{-1}) + (1-\tilde{\gamma})^2\tau_z^{-1} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \tag{F.3}$$

where M_{11} is 3×3 (making the distinction between what is in the information set of the agent and what is not). Similarly, let $m = (m_1, m_2)$, with m_1 being 3×1.

[16]Note that:

$$Var(P^{Mis}) = \tilde{\gamma}^2(\tau_0^{-1}+\tau_s^{-1}) + (1-\tilde{\gamma})^2\tau_z^{-1}.$$

While the covariances are given by:

$$Cov(P^{Mis}, v) = \tilde{\gamma}\tau_0^{-1},$$
$$Cov(P^{Mis}, s) = \tilde{\gamma}(\tau_0^{-1}+\tau_s^{-1}),$$
$$Cov(P^{Mis}, z) = (1-\tilde{\gamma})\tau_z^{-1}.$$

Then $(v, s, z)|P$ is jointly normal, with mean m^*, and covariance matrix M^*, where:

$$m^* = m_1 + M_{12}M_{22}^{-1}(P - m_2) \tag{F.4}$$

$$M^* = M_{11} - M_{12}M_{22}^{-1}M_{21}. \tag{F.5}$$

The first element in m^* and in M^* characterize uninformed agents' belief about the fundamental value of the asset, given their mapping: We can then write uninformed agents beliefs as follows:

$$\mathbb{E}[v|\mathcal{I}_U] = \mu_0 + \frac{\tau_s/\tilde{\gamma}}{\tau_0 + \tau_s + \left(\frac{1-\tilde{\gamma}}{\tilde{\gamma}}\right)^2 \frac{\tau_0\tau_s}{\tau_z}}(P - \mu_0 + (1-\tilde{\gamma})A\bar{z}\tau_0^{-1}) \tag{F.6}$$

$$= \mu_0 + \frac{g}{\tilde{\gamma}}(P - \mu_0 + (1-\tilde{\gamma})A\bar{z}\tau_0^{-1}) \tag{F.7}$$

$$\text{Var}[v|\mathcal{I}_U] = \tau_0^{-1} - \frac{\tau_s\tau_0^{-1}}{\tau_0 + \tau_s + \left(\frac{1-\tilde{\gamma}}{\tilde{\gamma}}\right)^2 \frac{\tau_0\tau_s}{\tau_z}} = \tau_0^{-1}(1-g) \tag{F.8}$$

where g is the Kalman gain:[17]

$$g = \frac{\tau_s}{\tau_0 + \tau_s + \left(\frac{1-\tilde{\gamma}}{\tilde{\gamma}}\right)^2 \frac{\tau_0\tau_s}{\tau_z}} \tag{F.9}$$

Putting the above components together, U agents' demand function is given by:[18]

$$X_U = \frac{\mu_0 + \frac{g}{\tilde{\gamma}}\left(P - \mu_0 + (1-\tilde{\gamma})A\bar{z}\tau_0^{-1}\right) - P}{A\tau_0^{-1}(1-g)}. \tag{F.10}$$

Moreover, I agents' demand function is simply given by:

$$X_I = \frac{\mu_0 + \frac{\tau_s}{\tau_s+\tau_0}(s - \mu_0) - P}{A(\tau_s + \tau_0)^{-1}} \tag{F.11}$$

As in our baseline case, the equilibrium price must solve the following market clearing

[17] When $\tau_z = \infty$, we recover the Kalman gain with fixed supply: $\tau_s/(\tau_0 + \tau_s)$.

[18] The algebra is less straightforward than in our baseline case, because PET agents are now using a lower precision than informed agents: their inference is weakened by the noisy supply.

condition, $z = \phi X_I + (1-\phi)X_U$:

$$z = \phi\left(\frac{\mu_0 + \frac{\tau_s}{\tau_s+\tau_0}(s-\mu_0) - P}{A(\tau_s+\tau_0)^{-1}}\right) + (1-\phi)\left(\frac{\mu_0 + \frac{g}{\tilde{\gamma}}\left(P - \mu_0 + (1-\tilde{\gamma})A\bar{z}\tau_0^{-1}\right) - P}{A\tau_0^{-1}(1-g)}\right) \quad \text{(F.12)}$$

Before solving for the equilibrium price, notice that, just as in Section 2.2, prices still play a dual role: their informational role gives rise to strategic complementarities, while their scarcity role gives rise to strategic substitutabilities. The interaction of these two forces determines whether the equilibrium is stable, as is clear from the excess demand function:

$$X_{TOT} - z = \frac{1}{A}\left(\underbrace{\frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\tilde{\gamma}}}_{\text{information role}} \underbrace{- \frac{\phi}{(\tau_0+\tau_s)^{-1}} - \frac{(1-\phi)}{\tau_0^{-1}(1-g)}}_{\text{scarcity role}}\right)P + constants \quad \text{(F.13)}$$

As in Section 2, the complementarities introduced by the informational role of prices give rise to a two-way feedback effect between outcomes and beliefs. The strength of this feedback effect is now given by:

$$\text{strength of feedback effect } = \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\tilde{\gamma}}, \quad \text{(F.14)}$$

and this is increasing in the influence that uninformed agents have on prices $\left(\frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\right)$, and decreasing in the sensitivity of the misspecified mapping $\tilde{\gamma}$.[19] When this feedback effect is strong enough, the aggregate excess demand function becomes upward sloping and the equilibrium unstable, just as we saw in Proposition 3. In particular, (F.13) shows that the equilibrium is stable if and only if:

$$\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} > \frac{(1-\phi)g}{\tau_0^{-1}(1-g)\tilde{\gamma}} \quad \text{(F.15)}$$

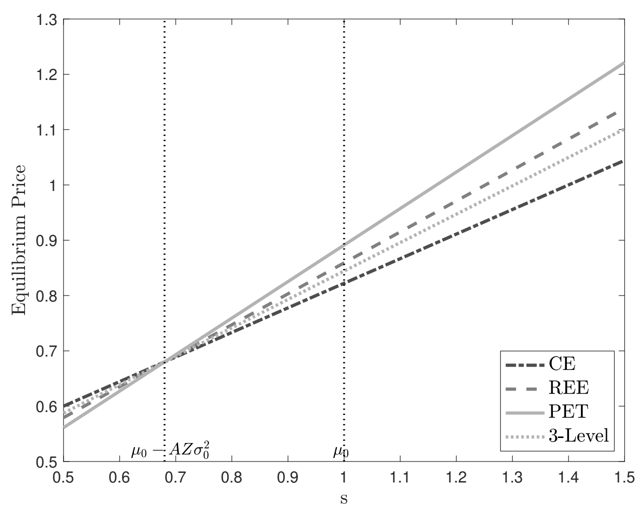
[19]Notice that increasing g has two effects: first, it reduces the conditional variance of uninformed agents, which makes them less risk averse, and this effect strengthens both the scarcity role and the informational role of prices in equal proportions. Second, it increases the weight that uninformed agents put on prices when performing Bayesian updating, and this increases the strength of the informational role. Therefore, the net effect of an increase in g is to strengthen the informational role relative to the scarcity role, and this contributed to instability. Moreover, as in Section 3, the strength of the feedback effect is decreasing in $\tilde{\gamma}$, so that the insights we uncovered in the fully revealing case are preserved.

We can now solve for the equilibrium price:

$$P = \frac{\phi\tau_0 + \frac{(1-\phi)\left(1-\frac{g}{\tilde{\gamma}}\right)}{\tau_0^{-1}(1-g)}}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\tilde{\gamma}}}\mu_0 + \frac{\phi\tau_s}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\tilde{\gamma}}}s$$
$$+ \frac{\frac{(1-\phi)g(1-\tilde{\gamma})}{(1-g)\tilde{\gamma}}}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\tilde{\gamma}}}A\bar{z} - \frac{1}{\frac{\phi}{(\tau_0+\tau_s)^{-1}} + \frac{(1-\phi)}{\tau_0^{-1}(1-g)} - \frac{(1-\phi)g}{\tau_0^{-1}(1-g)}\frac{1}{\tilde{\gamma}}}Az \quad \text{(F.16)}$$

In Figure 7, we plot this price function against the signal informed agents receive, s, together with the equilibrium price function of the CE and REE in this setup. Our benchmark results are preserved: PET features over-reaction, and lies outside the convex set of the CE and REE prices.[20] As in Section D, we can extend this argument to allow for $K-$level thinking. Figure 7 plots the equilibrium price for $3-$level thinking and we see that this once again delivers under-reaction. Overall results are qualitatively and quantitatively similar to our fully revealing benchmark. For the sake of clarity, and for ease of exposition, we focus on the fully revealing case in the main text.

Figure 7: Comparing Equilibrium Prices as we allow for rationality of Higher Order Beliefs in the Partially Revealing Case. This Figure plots the equilibrium prices which arise for different levels of $K-$thinking as we vary the value of the true signal, s. The CE is $1-$level thinking, PET is $2-$level thinking, and REE is $\infty-$level thinking. Here, ϕ is fixed at 0.6, $\tau_0^{-1} = 0.16$, $\tau_0^{-1} = 0.12$, $\mu_0 = 1$, $\bar{Z} = 1$, $\tau_z^{-1} = 0.1$, and $A = 2$.



Finally, notice how in this particular setup the PET equilibrium concept is much easier

[20]The fully cursed equilibrium (CE) benchmark corresponds to the case where no trader learns information from prices, and all traders trade on their private information alone.

to solve than the REE. Indeed, to solve for the REE one needs to postulate a price function, use Bayesian updating to find uninformed agents' demands conditional on the price function, and then work through market clearing to obtain the equilibrium price function. After this, agents still need to solve a fixed-point problem (that is far from trivial) where they equate the postulated price functions to the equilibrium price function. In this respect, PET agents are solving a much simpler problem, which requires far fewer cognitive skills than the REE.[21].

G Private signals

In this section we consider an alternative setup where agents are ex-ante symmetric, and all receive a different stochastic private signal. The price aggregates all private information, and agents infer the information received by others by conditioning on the equilibrium price. Specifically, there is a continuum of agents of measure 1, denoted by $i \in [0, 1]$. Supply is stochastic as in Appendix F. Each agent i receives a signal:

$$s_i = v + \epsilon_i \tag{G.1}$$

with $\epsilon_i \sim^{i.i.d.} \mathcal{N}(0, \tau_s^{-1})$. By the law of large numbers, the average signal received by the population corresponds to the period-1 payoff of the asset, $\int_0^1 s_i di = v$.

The misspecified model of the world corresponds to the case where all agents trade on their private information alone (the CE benchmark), which yields the following misspecified price function:

$$P^{Mis} = \tilde{\gamma} v + (1 - \tilde{\gamma})(\mu_0 - Az\tau_0^{-1}), \tag{G.2}$$

with $\tilde{\gamma} = \tau_s/(\tau_0 + \tau_s)$. All agents condition on s_i and P to update their prior. As previously, we start by constructing the variance-covariance structure:

$$Cov(v, s_i) = \tau_0^{-1}, \tag{G.3}$$

$$Cov(P^{Mis}, s_i) = \tilde{\gamma}\tau_0^{-1}. \tag{G.4}$$

[21]Computationally, the PET equilibrium is the first-step in solving the fixed-point process of the REE.

Which gives the following covariance matrix for the triplet (v, s_i, P):

$$M = \begin{pmatrix} \tau_0^{-1} & \tau_0^{-1} & \tilde{\gamma}\tau_0^{-1} \\ \tau_0^{-1} & \tau_0^{-1} + \tau_s^{-1} & \tilde{\gamma}\tau_0^{-1} \\ \tilde{\gamma}\tau_0^{-1} & \tilde{\gamma}\tau_0^{-1} & \tilde{\gamma}^2\tau_0^{-1} + (1-\tilde{\gamma})^2\tau_z^{-1} \end{pmatrix}. \tag{G.5}$$

We now need to define two Kalman gain coefficients, as all agents condition both on the price and on the private signal they receive:

$$g_s = \frac{(1-\tilde{\gamma})^2\tau_s}{\tilde{\gamma}^2\tau_z + (1-\tilde{\gamma})^2(\tau_0+\tau_s)}, \tag{G.6}$$

$$g_v = \frac{\tilde{\gamma}^2\tau_z}{\tilde{\gamma}^2\tau_z + (1-\tilde{\gamma})^2(\tau_0+\tau_s)}. \tag{G.7}$$

Therefore, agent's posterior beliefs of the fundamental value of the asset are normal, and characterized by the following posterior mean and variance:

$$E[v|s_i, P] = \mu_0 + g_s(s_i - \mu_0) + g_v\left(\frac{P - \mu_0 - (1-\tilde{\gamma})A\bar{z}\tau_0^{-1}}{\tilde{\gamma}}\right) \tag{G.8}$$

$$Var[v|s_i, P] = \tau_0^{-1}(1 - g_s - g_v). \tag{G.9}$$

Individual i's demand function is then given by:

$$X_i = \frac{\mu_0 + g_s(s_i - \mu_0) + g_v\left(\frac{P-\mu_0+(1-\tilde{\gamma})A\bar{z}\tau_0^{-1}}{\tilde{\gamma}}\right) - P}{A\tau_0^{-1}(1-g_s-g_v)}. \tag{G.10}$$

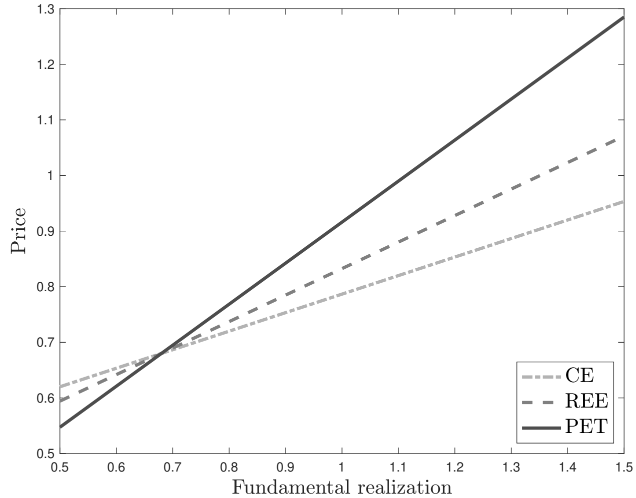
We can now impose market clearing and use the law of large numbers to aggregate private signals. This leads to the following equilibrium price expression:

$$P = \frac{1 - g_s - g_v/\tilde{\gamma}}{1-\frac{g_v}{\tilde{\gamma}}}\mu_0 + \frac{g_s}{1-\frac{g_v}{\tilde{\gamma}}}v - \frac{(1-g_s-g_v)}{1-\frac{g_v}{\tilde{\gamma}}}Az\tau_0^{-1} - \frac{g_v\frac{1-\tilde{\gamma}}{\tilde{\gamma}}}{1-\frac{g_v}{\tilde{\gamma}}}A\bar{z}\tau_0^{-1} \tag{G.11}$$

Figure 8 displays how the equilibrium price varies with the value of the fundamental. The results are qualitatively and quantitatively similar to our baseline framework.

Notice how the price sensitivity to the fundamental realization in (G.11) has exactly the

Figure 8: Comparing Equilibrium Prices with Symmetric Private Signals. This Figure plots the equilibrium prices as we vary the value of the fundamental realization, v. Here, $\tau_0^{-1} = 0.16$, $\tau_s^{-1} = 0.32$, and $\tau_z^{-1} = 0.95$ so that the condition which guarantees that the equilibrium PET price is stable is satisfied.



same form as the sensitivity of the price to new information in the general case with model misspecification in (46). With private signals, the feedback effect between outcomes and beliefs is now represented by the ratio $g_v/\tilde{\gamma}$, since g_v pertains to how agents incorporate into prices the information they get by learning from prices – the exact counterpart of $\hat{\beta}$ in our main framework. As such, all the insights we uncovered in the main text remain valid here. We conclude with a remark on how the volatility of supply relates to the stability of the equilibrium.

Corollary 5. *In this setup with symmetric agents and stochastic supply, the equilibrium is stable, and the price is increasing in the fundamental, if and only if:*

$$\tilde{\gamma} > g_v \iff \tau_z < \frac{\tau_0^2}{\tau_s} \tag{G.12}$$

In other words, once we allow for a symmetric information structure, the stochastic supply needs to be volatile enough for the equilibrium to be stable. Intuitively, when prices are close to fully revealing, the signal agents can recover from prices is infinitely more informative than their own private signal. Therefore, in this extreme case, agents do not use their own signal in their demand function. However, if all agents act in this way, no private signal is incorporated into prices, and the resulting price becomes completely uninformative. Since

PET agents fail to realize that all other agents are also learning from prices (and therefore that they are not incorporating their own private signal in them), they treat the price as informative even when it is not, and this contributes to unstable outcomes.

References

Angeletos, G.-M. and Z. Huo (2021). Myopia and anchoring. *American Economic Review 111*(4), 1166–1200.

Angeletos, G.-M. and C. Lian (2017). Dampening general equilibrium: From micro to macro. Technical report, National Bureau of Economic Research.

Angeletos, G.-M. and C. Lian (2018). Forward guidance without common knowledge. *American Economic Review 108*(9), 2477–2512.

Angeletos, G.-M. and K. A. Sastry (2020). Managing expectations: Instruments vs. targets. Technical report, Working Paper.

Bastianello, F. and P. Fontanier (2023). Partial Equilibrium Thinking, Extrapolation, and Bubbles. *Working Paper*.

Bohren, J. A. and D. N. Hauser (2021). Learning with heterogeneous misspecified models: Characterization and robustness. *Econometrica 89*(6), 3025–3077.

Bordalo, P., N. Gennaioli, and A. Shleifer (2018). Diagnostic expectations and credit cycles. *The Journal of Finance 73*(1), 199–227.

Costa-Gomes, M., V. P. Crawford, and B. Broseta (2001). Cognition and behavior in normal-form games: An experimental study. *Econometrica 69*(5), 1193–1235.

Costa-Gomes, M. A. and V. P. Crawford (2006). Cognition and behavior in two-person guessing games: An experimental study. *American Economic Review 96*(5), 1737–1768.

Crawford, V. P., M. A. Costa-Gomes, and N. Iriberri (2013). Structural models of nonequilibrium strategic thinking: Theory, evidence, and applications. *Journal of Economic Literature 51*(1), 5–62.

Diamond, D. W. and R. E. Verrecchia (1981). Information aggregation in a noisy rational expectations economy. *The Journal of Financial economics 9*(3), 221–235.

Frick, M., R. Iijima, and Y. Ishii (2019). Dispersed behavior and perceptions in assortative societies.

Gagnon-Bartsch, T. (2016). Taste projection in models of social learning. *Working Paper*.

Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus (2020). Five facts about beliefs and portfolios. *NBER Working Paper*.

Odean, T. (1998). Volume, volatility, price, and profit when all traders are above average. *The journal of finance 53*(6), 1887–1934.